Masses of Heavy Hadrons

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February 1, 2008

Abstract

An estimate has been made of the masses of heavy hadrons in nonrelativistic quark model, which includes spin and flavor-dependent hyperfine splitting for two quarks. The effect of variation of the wavefunction value at origin i.e. $|\psi(0)|^2$ and the strong coupling constant α_s, with flavor, has also been included in calculating the mass values.

Keywords: Mass, heavy baryons, heavy mesons.

PACS No.: 14.20.Kp, 14.40.Jz.

1 Introduction

A lot of data are available on the masses of mesons but the masses of most baryons have not been measured yet. Recently predictions about the heavy baryon mass spectrum have become a subject of increasing interest due to the current experimental activity of several groups at CERN, Fermilab and the Cornell Electron Storage Ring (CESR), aimed at the discovery of the baryons so far absent from the baryon summary table [1]. The copious production of heavy quarks at LEP, Fermilab Tevatron, CERN LHC and B factories, open for study the rich spectroscopy of heavy hadrons. So, a plausible theoretical prediction for the baryon mass spectrum becomes a guide for experimentalists.

Several models [2-10] have been used to evaluate the heavy baryon mass spectrum. The nonrelativistic quark model (NRQM), simple and economic one, has been able to explain very nicely the mass spectrum of light baryons and mesons. Many workers [3,4] have studied the masses of heavy hadrons in NRQM. Long time ago, Singh and Khanna [3], ignoring the effect of variation of strong coupling constant α_s and $|\psi(0)|^2$ (the wave function value at origin) with flavor, had estimated the masses of heavy hadrons using the NRQM with the inclusion of spin and flavor-dependent hyperfine interaction between two quarks and between a quark and an antiquark. They assume $|\psi(0)|^2$ scale for the heavy baryons to be the same as that of hyperons. However, since $|\psi(0)|^2$ is a dimensional quantity it may be incorrect to ignore its variation with flavor. Evidence to corroborate this has been found in the quark model [11-13] as well as in the lattice calculation [14]. Moreover, α_s is also scale dependent, so it will be worthwhile to include its variation with the mass scale.

In this paper, we calculate the masses of heavy hadrons in NRQM with the inclusion of variation of $|\psi(0)|^2$ and α_s with flavor.

2 Mass Formulae

The mass of a heavy hadron is assumed to arise from the constituent quark masses plus the two-body hyperfine interaction energies for a meson and a baryon. Without taking into consideration the variation of $|\psi(0)|^2$ and α_s with flavor, hadron masses, thus, can be expressed as [3],

$$M_{\text{meson}} = a_m + m_i + m_j + b m_0^2 \frac{\mathbf{s_i.s_j}}{m_i m_j}, \tag{1}$$

$$M_{\rm baryon} = a_b + \sum_i m_i + \frac{bm_0^2}{3}\sum_{i<j}\frac{\mathbf{s_i.s_j}}{m_i m_j}, \tag{2}$$

where a_m and a_b are the parameters with the dimension of mass and m_i (m_j) are the masses of respective quarks (antiquarks). m_0 is a scale factor which is taken to be the mass of the lightest quark i.e. $m_0 = m_u$. 'b', having the dimensions of mass, is a parameter which includes the long distance effects i.e. it takes care of the value of the wavefunction at origin, $|\psi(0)|^2$, along with the value of strong coupling constant α_s and is usually taken to be the same for all hadrons, irrespective of the quark flavor.

Now, if the dependence of $|\psi(0)|^2$ and α_s on flavor is taken into account, eqs. (1) and (2) modify to

$$M_{\rm meson} = a_m + m_i + m_j + b_{ij} m_0^2 \frac{\mathbf{s_i.s_j}}{m_i m_j}, \tag{3}$$

$$M_{\rm baryon} = a_b + \sum_i m_i + \frac{m_0^2}{3}\sum_{i<j} b_{ij}\frac{\mathbf{s_i.s_j}}{m_i m_j}. \tag{4}$$

Here, the scale factor, b_{ij}, instead of having a constant value for all flavors, is a variable and thus, has different values for different quark pairs. Hence, b_{ij} is the parameter which takes into consideration the flavor dependence of $|\psi(0)|^2$ and α_s, so that one has,

$$b_{ij} \propto |\psi_{ij}(0)|^2 \ \alpha_s(\mu). \tag{5}$$

The value of b_{ij} is taken to be same in the light quark sector i.e. for any of the u, d and s quark pairs and it varies as one goes to c-sector and then to b-sector.

3 Estimation of $|\psi(0)|^2$ and $\alpha_s(\mu)$

3.1 $|\psi(0)|^2$

The flavor dependence reflected in the scale factor b_{ij} corresponding to the spatial matrix element, is due to the long distance QCD effects. Evaluation of $|\psi(0)|^2$ is as yet uncertain for baryons and more complicated, because unlike the mesons, these are three-body systems. Infact, the heavy baryons may provide a good way of testing the flavor dependence of the confinement forces. The absence of an exact dynamical theory of low energy interactions between quarks limits our evaluation of $|\psi(0)|^2$ from first principles. However, a naive estimate for the scale parameter may be obtained using the hyperfine splitting (hfs) [2],

$$\Delta E_{\rm hfs} = \frac{16\pi\alpha_s}{9m_i m_j}|\psi(0)|^2 \mathbf{s_i.s_j}, \tag{6}$$

The experimental hyperfine splittings thus, may provide a reliable measure of the wave-function at origin of the ground state baryons the value of which is needed in the lifetime estimates.

Many workers have tried to estimate the value of $|\psi(0)|^2$ for different quark flavors using different techniques [13-18]. H. Y. Cheng [15], using formula (5) has found the values respectively, for c and b sectors to be

$$|\psi_{cu}(0)|^2 = 1.5 \times 10^{-2}\ \mathrm{GeV}^3 \tag{7}$$

and

$$|\psi_{bu}(0)|^2 = 2.5 \times 10^{-2}\ \mathrm{GeV}^3. \tag{8}$$

Körner and Siebert [16], from a fit to hyperfine splitting, have estimated the value of $|\psi(0)|^2$ for c-sector to be nearly 1.0×10^{-2} GeV^3. As mentioned in ref. [16], NRQM with a funnel type potential [17] and electromagnetic mass differences in the hyperfine splitting formula [18] also predict the values similar to that found by them. Values of the scale parameter $|\psi(0)|^2$ for different quark pairs have also been estimated in lattice approach and have been quoted in ref. [14].

Uppal and Verma [13], using eqn. (5) have made an estimate of the ratio $\frac{|\psi(0)|^2_c}{|\psi(0)|^2_s}$, which comes out to be approximately 2.83. We have also tried to estimate the wavefunction value at origin in terms of the ratios, $|\psi_{cu}|^2/|\psi_{su}|^2$ and $|\psi_{bu}|^2/|\psi_{su}|^2$, respectively, for c and b sectors.

3.2 $\alpha_s(\mu)$

The QCD coupling constant $\alpha_s(\mu)$ at any renormalization scale can be calculated from $\alpha_s(m_Z) = 0.117$ via

$$\alpha_s(\mu) = \frac{\alpha_s(m_Z)}{1 - (11 - \frac{2}{3}n_f)[\alpha_s(m_Z)/2\pi]ln(m_Z/\mu)}, \tag{9}$$

and one has,

$$\alpha_s(m_c) = 0.31 \quad \text{and} \quad \alpha_s(m_b) = 0.20 \tag{10}$$

4 Numerical Estimate for Masses

4.1 Heavy Baryons

Choosing the set of parameters (in MeV) to be $a_b = 205,\ m_0 = m_u = m_d = 310,\ m_s = 450,\ m_c = 1620,\ m_b = 4976,\ b_{qq'} = 640,\ b_{qc} = 736$ and $b_{qb} = 1600$, with q and $q' = u,\ d,\ s$, the masses of heavy baryons are calculated. Following the usual convention that a particle symbol represents its mass, the results for c-sector are displayed in Table 1 whereas Table 2 contains b-baryon masses. The mass values are in good agreement with the experimental values, wherever available. For comparison, masses estimated by other techniques have also been included in respective Tables 1 and 2.

Predictions have also been made for the masses of doubly heavy baryons using $|\psi_{qc}(0)|^2 = |\psi_{cc}(0)|^2$ and $|\psi_{qb}(0)|^2 = |\psi_{hb}(0)|^2$, with $h = b$ or c. Although it may not be an appropriate assumption, as no data are available in this sector, we can still make this choice. Moreover, when a particle contains two heavy quarks, in the heavy quark mass limit, the corresponding hyperfine splitting term ($\propto\ 1/m_i m_j$) will give negligible contribution to the overall mass and hence will be less significant. Predictions made on this assumption along with those made by other workers, have been given in Tables 1-3.

4.2 Heavy Mesons

Many different sets [8,19-29] of constituent quark masses to evaluate the masses of heavy mesons, are used in the literature, most of them obtained from fits to spectroscopic data. The values (in MeV) of various parameters used by us are: $a_m = 82,\ m_0 = m_u = m_d = 310,\ m_s = 400,\ m_c = 1578,\ m_b = 4920,\ b_{qq'} = 645,\ b_{qc} = 684$ and $b_{qb} = 755$, with q and $q' = u,\ d,\ s$. Mass values thus obtained are displayed in Table 4 and are in nice agreement with the experimental values.

It is interesting to note that the quark masses which give a best fit to the baryons are a little higher (except for the mass of u quark) than those which lead to a best fit to the mesons. Because these quark masses are constituent quark masses, there are no theoretical reasons why the masses determined from the baryons should coincide exactly with those determined from the mesons. If we insist that a single set of quark masses hold for both baryons and mesons, and vary these masses, the overall best fit to the hadron data is significantly poorer and our predictions have greater errors.

5 Discussion

At present, many of the charm baryon masses are known experimentally, whereas in b-sector, only the mass of one b-baryon is known accurately. The charm baryon masses (in MeV) measured till date are [1]

$$\begin{aligned}
\Lambda_c &= 2285, \\
\Sigma_c &= 2453 \pm 1, \\
\Xi_c &= 2468 \pm 2, \\
\Omega_c &= 2704 \pm 4, \\
\Sigma_c^* &= 2521 \pm 4, \\
\Xi_c^* &= 2644 \pm 2.
\end{aligned} \tag{11}$$

In 1994, WA89 Collaboration [30] had reported the observation of $\Xi_c' = 2563 \pm 15$ MeV. Recent calculations by many workers [8-10,31-34] consistently predict the mass of Ξ_c' to be around 2580 MeV. Also, the preliminary CLEO results on the $\Xi_c' - \Xi_c$ mass difference from $\Xi_c^{+\prime} \to \Xi_c^+\gamma$, $\Xi_c^{0\prime} \to \Xi_c^0\gamma$ decay [35] give us:

$$\begin{aligned}
\Xi_c^{+\prime} - \Xi_c^{+} &= 107.8 \pm 1.7 \pm 2.5\text{MeV}, \\
\Xi_c^{0\prime} - \Xi_c^{0} &= 107.0 \pm 1.4 \pm 2.5\text{MeV}.
\end{aligned} \tag{12}$$

With $\Xi_c = 2468 \pm 2$ MeV, these results lead to

$$\Xi_c' = 2575 \pm 5\text{MeV}. \tag{13}$$

Our prediction for Ξ_c' (2580 MeV) is in nice agreement with this experimental observation as well as with the predictions made by other authors. Also the mass difference $\Xi_c' - \Xi_c$, using our mass values, comes out to be 107 MeV, which coincides very well with the experimentally measured value.

The Ω_c^* as well as double and triple charm baryons, have not yet been observed. Most of the theoretical calculations [7-10,31-34,36] predict the mass of Ω_c^* to be around 2770 MeV, whereas predictions for the doubly and triply charmed baryon masses are less definite. The mass value, as predicted by us, for Ω_c^* (2766 MeV) is consistent with the above mentioned theoretical predictions.

On the other hand, in the b-sector, only the mass of Λ_b [1] baryon is known well and masses of the rest of the b-baryons are yet to be measured. The mass of Λ_b baryon as

predicted by us is in nice agreement with the experimental value [1], whereas the rest of b-baryon masses predicted by us agree well with the predictions made by ref. [8], as can be seen from Table 2.

To have an estimate of the wavefunction value at origin, we make use of eqn. (5) i.e. $b_{ij} \propto |\psi_{ij}(0)|^2\ \alpha_s(\mu)$, so that one has

$$\frac{b_{cu}}{b_{su}} = \frac{|\psi_{cu}(0)|^2 \alpha_s(m_c)}{|\psi_{su}(0)|^2 \alpha_s(m_s)} \tag{14}$$

and

$$\frac{b_{bu}}{b_{su}} = \frac{|\psi_{bu}(0)|^2 \alpha_s(m_b)}{|\psi_{su}(0)|^2 \alpha_s(m_s)} \tag{15}$$

Using our preferred set of parameters, these ratios turn out to be

$$\frac{b_{cu}}{b_{su}} = 1.15 \tag{16}$$

and

$$\frac{b_{bu}}{b_{su}} = 2.5 \tag{17}$$

As the mass dependence of the strong coupling constant (eqn. 9) is known, an estimate of the ratios of the wave function values at origin (i.e. $|\psi_{cu}|^2/|\psi_{su}|^2$ and $|\psi_{bu}|^2/|\psi_{su}|^2$) can be made using eqs. (15) to (18):

$$\begin{aligned} |\psi_{cu}|^2/|\psi_{su}|^2 &= 3.3 \\ \text{and}\ |\psi_{bu}|^2/|\psi_{su}|^2 &= 11.3 \end{aligned} \tag{18}$$

The ratio for c-sector is little higher than the estimates of ref. [13] and that of lattice calculations [14]. It may be due to the fact that the ratio also depends on the strong coupling constant for light sector, whose value is not fixed. So different values of $\alpha_s(m_s)$ will lead to different values of these ratios. Only the experimental data in future will give us better insight in this arena.

Note that, irrespective of introducing the parameters concerning the variation of wavefunction at origin with flavor, certain independent relations and mass shifts introduced in ref. [3] are satisfied here also:

$$\Omega_b^* + \Sigma_b^* - 2\Xi_b^* = \Omega_c^* + \Sigma_c^* - 2\Xi_c^* = \Omega^* + \Sigma^* - 2\Xi^*, \tag{19}$$

$$(\Sigma_{cc}^* - \Sigma_c^*) = \Xi_{cc} - \Sigma_c, \tag{20}$$

$$(\Sigma_{bb}^* - \Sigma_b^*) = \Xi_{bb} - \Sigma_b, \tag{21}$$

$$\frac{(\Sigma_b^* - \Sigma_b)}{(B^* - B)} = \frac{(\Sigma_c^* - \Sigma_c)}{(D^* - D)} = \frac{(\Sigma^* - \Sigma)}{(K^* - K)} = \frac{\Delta - N}{\rho - \pi} = \frac{1}{2} \tag{22}$$

$$\frac{3}{2}\frac{\Sigma_b - \Lambda_b}{\Delta - N} + \frac{B^* - B}{\rho - \pi} = \frac{3}{2}\frac{\Sigma_c - \Lambda_c}{\Delta - N} + \frac{D^* - D}{\rho - \pi} = \frac{3}{2}\frac{\Sigma - \Lambda}{\Delta - N} + \frac{K^* - K}{\rho - \pi} = 1 \tag{23}$$

The predicted masses may have large errors, so it may be useful to have an idea of the mass difference from the lowest lying heavy baryon. In b-sector, one has (in MeV):

$$\begin{aligned}
\Sigma_b - \Lambda_b &= 180, \\
\Sigma_b^* - \Lambda_b &= 230, \\
\Xi_b - \Lambda_b &= 188, \\
\Xi_b' - \Lambda_b &= 302, \\
\Xi_b^* - \Lambda_b &= 343, \\
\Omega_b - \Lambda_b &= 436, \\
\Omega_b^* - \Lambda_b &= 469
\end{aligned} \tag{24}$$

To conclude, by allowing the variation of parameter b with flavor and introducing two more parameters, we are able to bring lot of masses of both baryons and mesons closer to the observed values.

6 Acknowledgement

HK thanks Prof. M. P. Khanna for useful discussions and Council of Scientific and Industrial Research, New Delhi, India, for fellowship.

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Table 1: Masses (in MeV) of charm baryons

Particle	Calculated Mass	Experimental Value	Ref. [3]	Ref. [8]	Ref. [34]
Λ_c	2285	2285	2267	2285±1	-
Σ_c	2451	2455	2436	2453±3	-
Ξ_c	2475	2470	2504	2468±3	-
Ξ_c'	2582	2580*	2602	2580±20	2569±6
Ω_c	2717	2704	2775	2710±30	-
Σc^*	2521	2521	2491	2520±20	-
Ξ_c^*	2642	2644	2646	2650±20	-
Ω_c^*	2766	-	2800	2770±30	2767±7
Ξ_{cc}	3714	-	3710	3660±70	3610±3
Ξ_{cc}^*	3786	-	3781	3740±70	3735±17
Ω_{cc}	3882	-	3865	3740±80	3804±8
Ω_{cc}^*	3914	-	3948	3820±80	3850±25

Table 2: Masses (in MeV) of beauty baryons

Particle	Calculated Mass	Experimental Value	Ref. [3]	Ref. [8]
Λ_b	5641	5641	5724	5620±40
Σ_b	5821	5814*	5729	5820±40
Ξ_b	5831	-	5783	5810±40
Ξ_b'	5949	-	5893	5950±40
Ω_b	6083	-	6065	6060±50
Σb^*	5871	5870*	5750	5850±40
Ξ_b^*	5991	-	5915	5980±40
Ω_b^*	6117	-	6079	6090±50
Ξ_{bb}	10434	-	10315	10340±100
Ξ_{bb}^*	10484	-	10321	10370±100
Ω_{bb}	10585	-	10494	10370±100
Ω_{bb}	10619	-	10321	10400±100

* as calculated from the mass differences.

Table 3: Masses (in MeV) of baryons containing two different heavy quarks i.e. c and b.

Particle	Ξ_{bc}	Ξ'_{bc}	Ξ^*_{bc}	Ω_{bc}	Ω'_{bc}	Ω^*_{bc}
Mass	7076	7102	7133	7227	7244	7266
Ref. [8]	6990±90	7040±90	7060±90	7060±90	7090±90	7120±90

Table 4: Masses (in MeV) of heavy mesons.

Particle	Calculated Mass	Experimental Value [1]
D	1869	1869
D^*	2003	2007
D_s	1982	1968
D_s^*	2086	2112
B	5276	5279±1.8
B^*	5324	5325±1.8
B_s	5374	5369±2
B_s^*	5411	-